

# CPW Resonator Modelling on GaAs Using the Mixed Potential Integral Equation

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## ABSTRACT

With the advance of coplanar waveguides in monolithic millimeter wave integrated circuits (M<sup>3</sup>ICs) accurate modelling of planar geometries is going to take on more and more significance. Fully electromagnetic models for arbitrary geometries are required in order to include effects such as dispersion, radiation and coupling. A coplanar waveguide (CPW) resonator on grounded GaAs substrate is presented and investigated numerically using a space-domain integral equation method. The numerical results are compared with on wafer measurements and show excellent agreement. The exact calculation of the input impedance as well as the verification of unwanted parasitic resonances is reported. The coupling capacitance, the most sensitive design parameter, is used to tune the circuit and therefore its dependence of the gap width is plotted additionally.

## INTRODUCTION

The use of coplanar structures in M<sup>3</sup>IC design allows the reduction of the geometric dimensions in the circuit plane irrespective from the substrate thickness and avoids the necessity of via holes [1]. Especially in the millimeter wave range coplanar circuit geometries are advantageous over microstrip geometries. The accurate analysis of CPW based circuits, however, requires three-dimensional electromagnetic field computation. Thus full-wave analysis tools have to be developed to simulate monolithic millimeter wave integrated circuits (MMICs). For the design of passive planar structures integral equation techniques like the Mixed Potential Integral Equation (MPIE) are well suited [2].

## MODELLING

The MPIE [3] is used to model numerically grounded single-layered coplanar waveguide structures in the frequency domain.

$$\mathbf{e}_z \times \mathbf{E}^e = Z_s \mathbf{J}_s + \mathbf{e}_z \times (j\omega \int_{S_0} \bar{G}_A \mathbf{J}_s dS' + \nabla \int_{S_0} G_V q_s dS') \quad (1)$$

The Green's functions of the Sommerfeld type,  $\bar{G}_A$  and  $G_V$  are derived analytically in the spectral domain and transformed numerically into space domain. The unknown electrical surface current distribution  $\mathbf{J}_s$  is expanded into subdomain basis functions and the Method of Moments (MoM) is applied to the planar structure under investigation. With the choice of rooftop-type basis functions for the current distribution  $\mathbf{J}_s$ , two-dimensional pulse functions for the electrical charge  $q_s$  and razor edge test functions along  $C_j$  we obtain for example for a subsection of the size  $2a \times b$  and orientation in  $x$ -direction the following test- and basis functions in Fig.1. The resulting equation system

$$\begin{pmatrix} \bar{C}^{xx} & \bar{C}^{xy} \\ \bar{C}^{yx} & \bar{C}^{yy} \end{pmatrix} \begin{pmatrix} \mathbf{I}_x \\ \mathbf{I}_y \end{pmatrix} = \frac{1}{jZ_0} \begin{pmatrix} \mathbf{V}_x^{(e)} \\ \mathbf{V}_y^{(e)} \end{pmatrix} \quad (2)$$

is set up for various frequencies and solved with an iterative method. The elements of the submatrices contain the reaction integrals of basis- and testfunctions and are given by Eq.(3) and Eq.(4).

$$\begin{aligned} C_{ij}^{xx} = & \frac{1}{k_0 a k_0 b} [\Gamma_V(\mathbf{r}_{xi}^+ | \mathbf{r}_{xj}^+) - \Gamma_V(\mathbf{r}_{xi}^+ | \mathbf{r}_{xj}^-) \\ & - \Gamma_V(\mathbf{r}_{xi}^- | \mathbf{r}_{xj}^+) + \Gamma_V(\mathbf{r}_{xi}^- | \mathbf{r}_{xj}^-)] \\ & - \frac{1}{k_0 b} \int_{C_{xi}} \Gamma_A^{xx}(\mathbf{r}' | \mathbf{r}_{xj}) k_0 dx' + j \frac{Z_s a}{Z_0 8b} (6\delta_{ij} + \delta_{i+1,j} + \delta_{i-1,j}) \quad (3) \end{aligned}$$

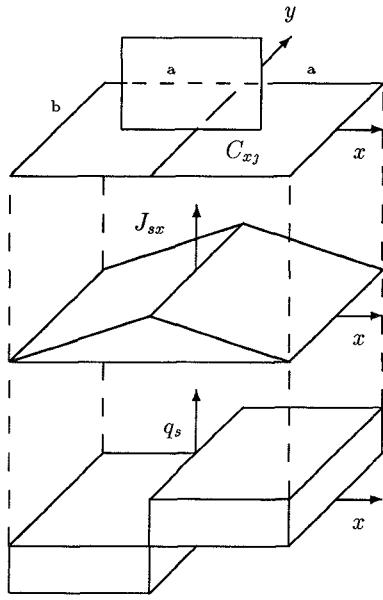


Figure 1: Subsectional test and basis function

$$C_{ij}^{xy} = \frac{1}{k_0 a k_0 b} [\Gamma_V(\mathbf{r}_{xi}^+ | \mathbf{r}_{yj}^+) - \Gamma_V(\mathbf{r}_{xi}^+ | \mathbf{r}_{yj}^-) - \Gamma_V(\mathbf{r}_{xi}^- | \mathbf{r}_{yj}^+) + \Gamma_V(\mathbf{r}_{xi}^- | \mathbf{r}_{yj}^-)] \quad (4)$$

Interchanging the pairs  $(x, y)$  and  $(a, b)$  we obtain the remaining matrix elements. The computed current distributions will then be used to calculate the input impedance and to detect the excited resonances respectively.

Concerning the geometrical shape of complex passive planar structures discretization usually requires the solution of large systems of linear equations that may exceed the storage of a computer. Because of numerical instabilities Gaussian elimination becomes intractable for higher frequencies. To overcome these drawbacks we use Conjugate Gradient Method (CGM) [4] for the numerical solution of the matrices and apply a mapping procedure to the matrix elements that allows a reduction in computational storage by about one dimension [5].

Taking advantage of the rotational and translational symmetry of the Green's functions in the transverse directions, one can map the whole MoM-matrix onto two rows of the matrix. Only as few matrix elements as absolutely necessary,

have to be addressed and thus the information of the quadratic system matrix can be stored in the computational memory of two rows. The numerical algorithm is fully vectorizable and therefore very suited to run on Cray computers. It extends the range of discretization drastically and allows to examine passive planar radiating structures with ten thousand and more subsections.

The convergence behaviour of the CG-algorithm depends on the parameters used to construct the matrix. Furthermore it is linked to the eigenvalue structure of the matrix, which itself is an approximation to the eigenvalue spectrum of the integral operator. Thus the convergence behaviour of the CGM can be used for indicating the adequacy of the discretization used to form the matrix. A familiarity with the performance of the CGM is helpful in identifying situations where the numerical modeling is inadequate to produce an accurate solution to the original equation.

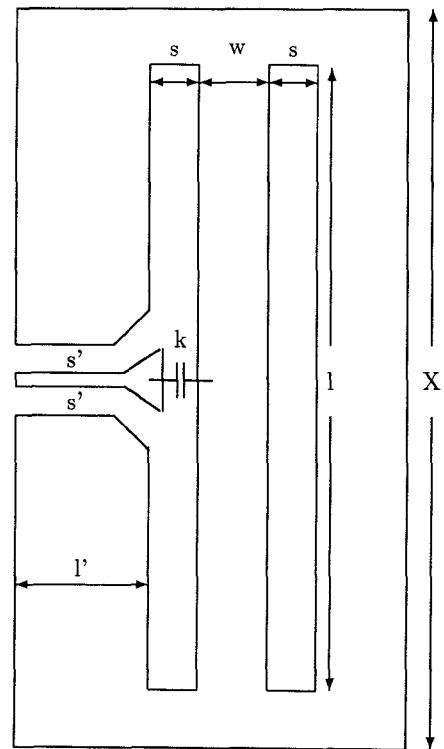


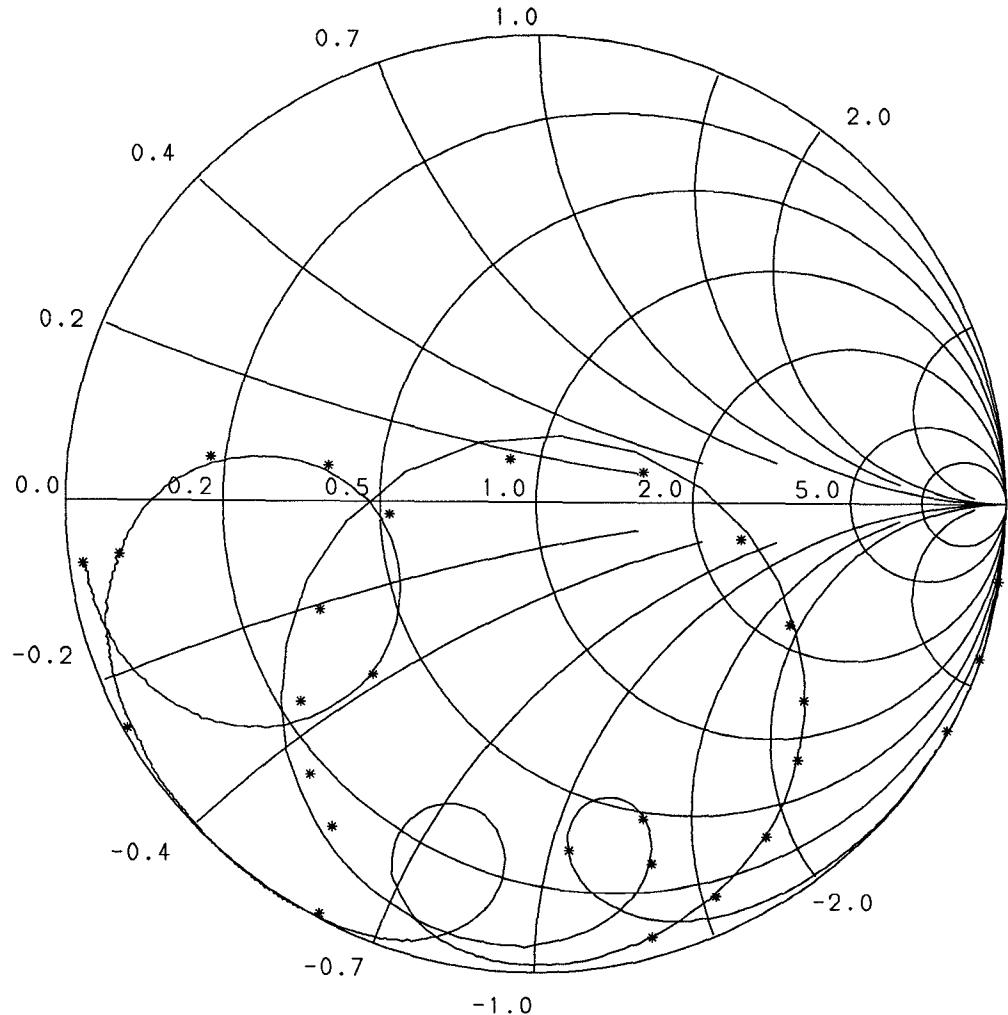
Figure 2: Layout of the CPW resonator

## DESIGN AND MEASUREMENT

We integrated several coplanar-fed gap-coupled coplanar waveguide resonators on grounded  $400\mu\text{m}$  thick GaAs substrate ( $\epsilon_r=12.9$ ,  $\tan \delta=6 \cdot 10^4$ ) (Fig.2). The planar resonator was designed to excite the even coplanar waveguide mode. With the length  $l=3.0\text{mm}$ , the width  $w=135.0\mu\text{m}$  and the slots  $s=67.5\mu\text{m}$  we obtain a strong CPW resonance at 18.5 GHz. A coplanar feeding line ends in a taper that couples capacitively the resonator through a gap. The gap spacing is tuned within the range of  $10\mu\text{m}$  to  $35\mu\text{m}$ . The passive circuit is located within a rectangular patch of the size  $X=3.600\text{mm}$ ,  $Y=1.095\text{mm}$  and the conductor thickness amounts to  $2\mu\text{m}$  (gold).

To numerically evaluate the input impedance of the CPW resonator the planar structure has to be discretized into equivalent rectangular subsections. Therefore the real geometry of the passive planar structure has to be replaced by a staircase-approximation. The simulation of the millimeter wave structure takes place within an equally spaced discretization grid of 7680 equivalent rectangular subsections.

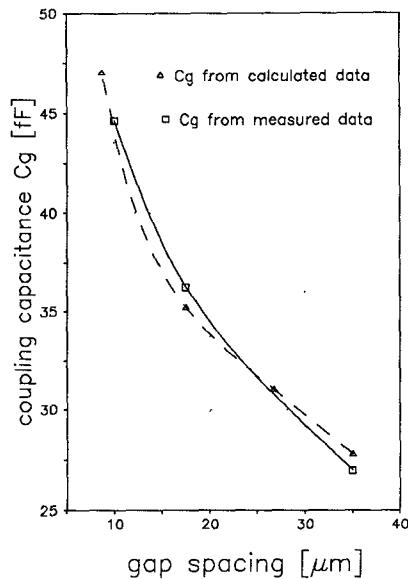
On wafer reflection measurements in the frequency range from 1 to 40 GHz were performed using an HP8510. The measured resonator structure show four resonances in Fig.4. The tightest coupled resonance is caused by the coplanar mode. The first resonance of 14.9 GHz and the last reson-



**Figure 4:** Results ( $Z_w=50\Omega$ ): - measurement (1GHz - 40GHz) and \* simulated data

ance of 36.5 GHz are due to the spurious capacities that couple into the propagating mode of the lower slotline. They can be successfully suppressed using airbriges or bondwires. The parasitic resonance represented by the third circle is supposed to be excited by the measurement setup.

To include the simulated and measured results into commercially available CAD tools a mixed lumped-distributed equivalent circuit topology is developed to model the frequency response.



**Figure 3:** Dependency of the coupling capacitance  $C_g$

The gap spacing is found to be the most sensitive part of the structure. This tuning parameter is modeled by a coupling capacitance  $C_g$  (Fig.3) and is responsible for the tightness of the coupling. The measured and computed data show excellent agreement.

## CONCLUSION

This paper reports the modelling of a  $\lambda/2$  coplanar resonator on grounded GaAs substrate. The input impedance of the planar structure is simulated by an integral equation technique in the space-domain, the Mixed Potential Integral

Equation. Results indicate very good matching and confirms the feasibility of the numerical techniques. With the usage of an equally spaced discretization grid one not only saves the maximum of computational storage for the numerical simulation but also can implement highly vectorized algorithms. The presented method is promising for applications in planar microwave circuits. The most sensitive parameter of the circuit, the capacitive gap spacing, is modelled by a coupling capacitance. Measured and calculated results show excellent agreement.

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